

Back to the Future: The Lee Wick Standard Model.

University of California, Berkeley, October, 2007

Mark B. Wise, California Institute of Technology

Outline

The Standard Model pg. 3

Hierarchy Problem. 4-6

Some Other References 7

Toy Model 8-14

Lee-Wick Gauge Bosons and the Hierarchy Problem 15-21

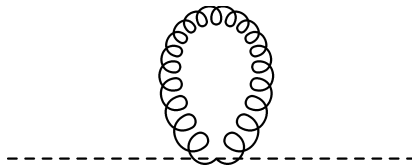
Fermions 22-23

Conclusion 24

Field	$SU(3)_c$	$SU(2)_L$	$U(1)_Y$	Lorentz
$Q_L^i = \begin{pmatrix} u_L^i \\ d_L^i \end{pmatrix}$	3	2	1/6	(1/2,0)
u_R^i	3	1	2/3	(0,1/2)
d_R^i	3	1	-1/3	(0, 1/2)
$L_L^i = \begin{pmatrix} \nu_L^i \\ e_L^i \end{pmatrix}$	1	2	-1/2	(1/2,0)
e_R^i	1	1	-1	(0,1/2)
$H = \begin{pmatrix} H^+ \\ H^0 \end{pmatrix}$	1	2	1/2	(0,0)

The Hierarchy Problem

- Motivation for new physics beyond weak scale, Hierarchy Problem. SM Higgs potential: $V(H) = \lambda(H^\dagger H - v^2/2)^2$ implies vacuum expectation value $H^0 = v/\sqrt{2}$ physical Higgs scalar mass $m_h \sim \sqrt{\lambda}v$. $v/M_{PL} \sim 10^{-17}$.
- One loop radiative corrections from coupling to W boson induce ; $\delta m_h^2 \sim g_2^2 \Lambda_c^2 / 16\pi^2$.



- If Λ_c is of order the Planck scale then $\delta m_h^2 \sim (10^{18} \text{GeV})^2$. To keep v small requires a delicate fine tuning between bare parameters and radiative corrections.
- Some of solutions proposed: Dynamical Symmetry Breaking (Technicolor), Low Energy Supersymmetry, Large Extra Dimensions, Warped Extra Dimensions, Anthropic Principle **A**.
- Lee-Wick Standard Model presented in a paper with Donal O'Connell and Benjamin Grinstein arXiv:0704.1845. It builds on ideas first presented in two papers, T. D. Lee and G. C. Wick, Nucl Phys B**9**, 209 (1969) and T.D. Lee and G. C. Wick, Phys. Rev. D**2**, 1033 (1970). Paradigm shift not causal.

**Congratulations Saul
Perlmutter on winning the
Nobel Prize**

Some Other References

1. Cutkosky, et. al., Nucl. Phys. B**12**, 281 (1969).
2. Nakanishi, Phys. Rev. D**3**, 811 (1971).
3. Lee and Wick, Phys. Rev. D**3** 1046 (1971).
4. Coleman, In “Erice 1969, Ettore Majorana School On Subnuclear Phenomena”, New York, 182 (1970).
5. Tomboulis, Phys. Lett. B**70**, 361 (1977); B**97**, 77 (1980).
6. Boulware and Gross, Nucl. Phys. B**233**, 1 (1984).
7. Jensen, et. al., Phys. Lett. B**309**, 119 (1993); 127 (1993).
8. Rizzo, arXiv:0704.3458 [hep-ph].
9. Espinosa, et. al. , arXiv:0705.1188 [hep-ph].
10. Wu and Zhong, arXiv:0705.3287 [hep-ph].
11. Dulaney and Wise, arXiv:0708.0567 [hep-ph].
12. Krauss, et. al., arXiv:0709.4054 [hep-ph].

A Toy Model

- Lagrange density

$$\mathcal{L}_{\text{hd}} = \frac{1}{2} \partial_\mu \hat{\phi} \partial^\mu \hat{\phi} - \frac{1}{2M^2} (\partial^2 \hat{\phi})^2 - \frac{1}{2} m^2 \hat{\phi}^2 - \frac{1}{3!} g \hat{\phi}^3,$$

propagator of $\hat{\phi}$

$$\hat{D}(p) = \frac{i}{p^2 - p^4/M^2 - m^2}.$$

$M \gg m$, poles at $p^2 \simeq m^2$ and also at $p^2 \simeq M^2$. More than one degree of freedom. Residue of $p^2 \simeq M^2$ wrong sign. Unstable.

- Auxiliary Lee Wick scalar field $\tilde{\phi}$. Write the theory as

$$\mathcal{L} = \frac{1}{2}\partial_\mu\hat{\phi}\partial^\mu\hat{\phi} - \frac{1}{2}m^2\hat{\phi}^2 - \tilde{\phi}\partial^2\hat{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{3!}g\hat{\phi}^3.$$

Removing $\tilde{\phi}$ from \mathcal{L} with their equations of motion reproduces \mathcal{L}_{hd} .

- Define $\phi = \hat{\phi} + \tilde{\phi}$ to remove $\tilde{\phi}$ - $\hat{\phi}$ kinetic mixing.

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}\partial_\mu\tilde{\phi}\partial^\mu\tilde{\phi} + \frac{1}{2}M^2\tilde{\phi}^2 - \frac{1}{2}m^2(\phi - \tilde{\phi})^2 - \frac{1}{3!}g(\phi - \tilde{\phi})^3.$$

Two kinds of scalar field: a normal scalar field ϕ and a new LW-field $\tilde{\phi}$. $\tilde{\phi}$ can decay to two ϕ 's. Lee Wick resonance, calculate its decay width from two-point function of $\tilde{\phi}$.

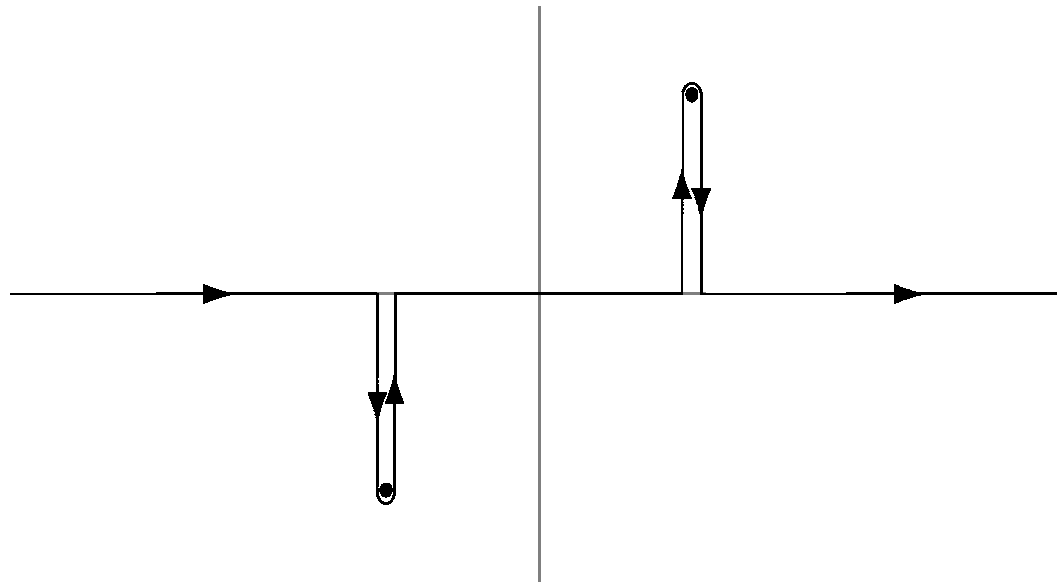
$$\text{---} \underset{LW}{\text{---}} \text{---} + \text{---} \underset{LW}{\text{---}} \bigcirc \text{---} \underset{LW}{\text{---}} \text{---} + \dots$$

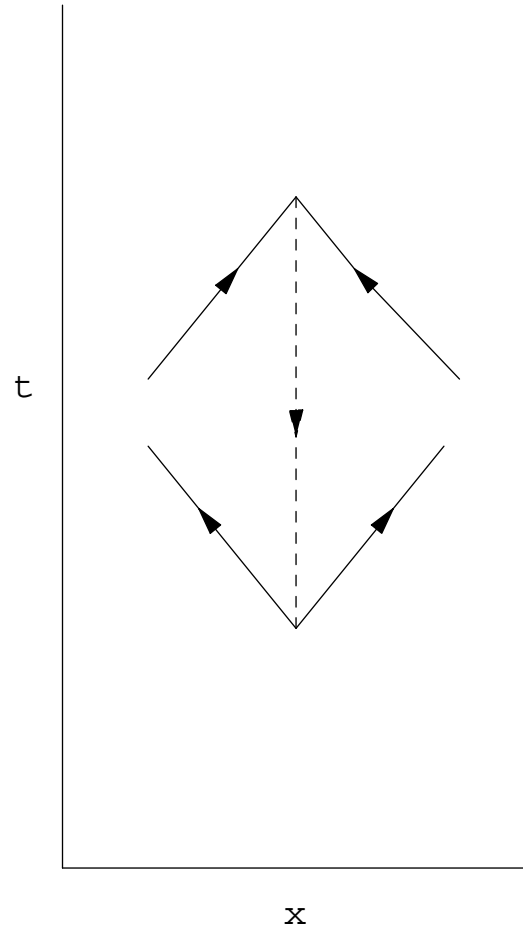
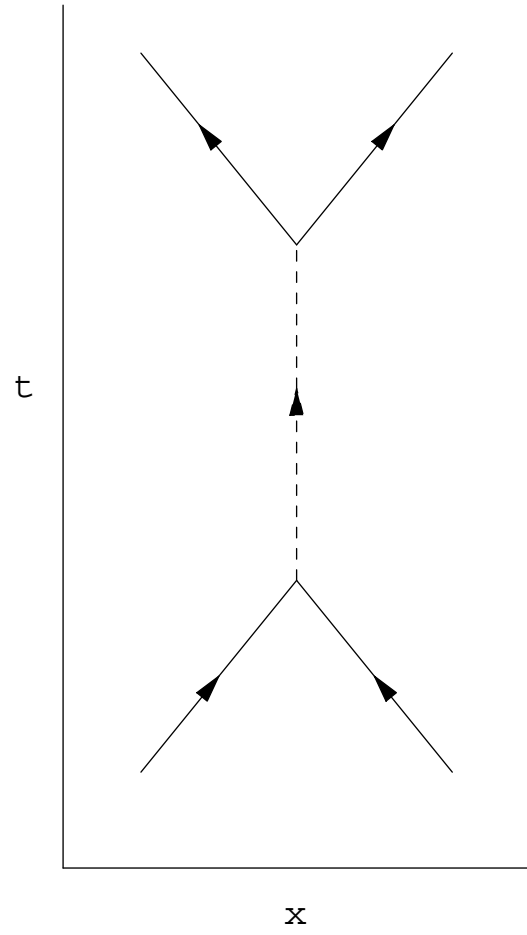
- Bubble sum,

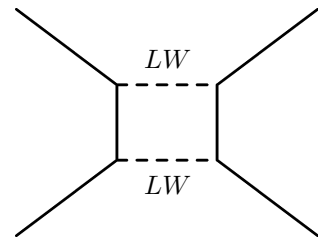
$$\begin{aligned}\tilde{D}(p) &= \frac{-i}{p^2 - M^2} + \frac{-i}{p^2 - M^2}(-i\Sigma(p^2))\frac{-i}{p^2 - M^2} + \dots \\ &= \frac{-i}{p^2 - M^2 + \Sigma(p^2)}.\end{aligned}$$

- $(-i\Sigma(p^2))$ same as usual but extra minus sign with each $(-i\Sigma(p^2))$. Width,

$$\Gamma = -\frac{g^2}{32\pi M} \sqrt{1 - \frac{4m^2}{M^2}}.$$







LW-Gauge Bosons and the Hierarchy Problem

- Higher derivative Lagrange density

$$\mathcal{L}_{\text{hd}} = -\frac{1}{2}\text{tr}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} + \frac{1}{M_A^2}\text{tr}\left(\hat{D}^\mu\hat{F}_{\mu\nu}\right)\left(\hat{D}^\lambda\hat{F}_{\lambda}{}^\nu\right),$$

where $\hat{F}_{\mu\nu} = \partial_\mu\hat{A}_\nu - \partial_\nu\hat{A}_\mu - ig[\hat{A}_\mu, \hat{A}_\nu]$, and $\hat{A}_\mu = \hat{A}_\mu^A T^A$ with T^A the generators of the gauge group G in the fundamental representation. Choose this higher derivative term because can remove it using LW-field \tilde{A}_μ .

-

$$\mathcal{L} = -\frac{1}{2}\text{tr}\hat{F}_{\mu\nu}\hat{F}^{\mu\nu} - M_A^2\text{tr}\tilde{A}_\mu\tilde{A}^\mu + 2\text{tr}\hat{F}_{\mu\nu}\hat{D}^\mu\tilde{A}^\nu,$$

where $\hat{D}_\mu\tilde{A}_\nu = \partial_\mu\tilde{A}_\nu - ig[\hat{A}_\mu, \tilde{A}_\nu]$.

- To diagonalize the kinetic terms, we introduce shifted fields defined by

$$\hat{A}_\mu = A_\mu + \tilde{A}_\mu.$$

The Lagrangian becomes

$$\begin{aligned} \mathcal{L} = & -\frac{1}{2}\text{tr}F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\text{tr}\left(D_\mu\tilde{A}_\nu - D_\nu\tilde{A}_\mu\right)\left(D^\mu\tilde{A}^\nu - D^\nu\tilde{A}^\mu\right) \\ & -ig\text{tr}\left([\tilde{A}_\mu, \tilde{A}_\nu]F^{\mu\nu}\right) - \frac{3}{2}g^2\text{tr}\left([\tilde{A}_\mu, \tilde{A}_\nu][\tilde{A}^\mu, \tilde{A}^\nu]\right) \\ & -4ig\text{tr}\left([\tilde{A}_\mu, \tilde{A}_\nu]D^\mu\tilde{A}^\nu\right) - M_A^2\text{tr}\left(\tilde{A}_\mu\tilde{A}^\mu\right). \end{aligned}$$

- Only dimension four operators but longitudinal massive vector bosons can cause bad high energy behavior. Standard model with no Higgs but mass terms for W-bosons $\mathcal{A}(LL \rightarrow LL) \sim g^2 E^2 / M^2$. Arises from $\epsilon_L(p) = (|\mathbf{p}|, E(p)\hat{\mathbf{p}})/M$.

- For Lee-Wick massive gauge bosons no bad high energy behavior of scattering amplitudes at tree level $\mathcal{A}(LL \rightarrow LL) \sim g^2 M^2/E^2$, $\mathcal{A}(LL \rightarrow LT) \sim g^2 M/E$, $\mathcal{A}(LL \rightarrow TT) \sim g^2$.
- To discuss hierarchy problem add scalar multiplet in fundamental representation

$$\mathcal{L}_{\text{hd}} = (\hat{D}_\mu \hat{\phi})^\dagger (\hat{D}^\mu \hat{\phi}) - \frac{1}{M_\phi^2} (\hat{D}_\mu \hat{D}^\mu \hat{\phi})^\dagger (\hat{D}_\nu \hat{D}^\nu \hat{\phi}).$$

- $\hat{\phi} = \phi - \tilde{\phi}$

$$\begin{aligned} \mathcal{L} = & (D_\mu \phi)^\dagger D^\mu \phi - (D_\mu \tilde{\phi})^\dagger D^\mu \tilde{\phi} + M_\phi^2 \tilde{\phi}^\dagger \tilde{\phi} \\ & + ig(D^\mu \phi)^\dagger \tilde{A}_\mu^A T^A \phi + g^2 \phi^\dagger \tilde{A}_\mu^A T^A \tilde{A}^{B\mu} T^B \phi - ig\phi^\dagger \tilde{A}_\mu^A T^A D^\mu \phi \\ & - ig(D^\mu \tilde{\phi})^\dagger \tilde{A}_\mu^A T^A \tilde{\phi} + ig\tilde{\phi}^\dagger \tilde{A}_\mu^A T^A D^\mu \tilde{\phi} - g^2 \tilde{\phi}^\dagger \tilde{A}_\mu^A T^A \tilde{A}^{B\mu} T^B \tilde{\phi}. \end{aligned}$$

- Work in higher derivative theory and fix gauge in usual way. Gauge boson propagator

$$\hat{D}_{\mu\nu}^{AB}(p) = \delta^{AB} \frac{-i}{p^2 - p^4/M_A^2} \left(\eta_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2} - \xi \frac{p_\mu p_\nu}{M_A^2} \right).$$

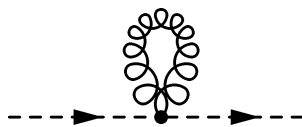
No quadratic divergence in theory in Landau $\xi = 0$ gauge;
 $d = 6 - 2L - E - E' - 2E_g$.

- Explicit one loop calculation in LW-version of theory with no higher derivative terms. Propagators for gauge bosons,

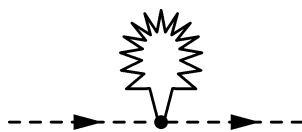
$$D_{\mu\nu}^{AB}(p) = -\delta^{AB} \frac{i}{p^2} \left(\eta_{\mu\nu} - (1 - \xi) \frac{p_\mu p_\nu}{p^2} \right),$$

while the propagator for the LW-vector field is

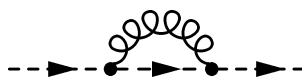
$$\tilde{D}_{\mu\nu}^{AB}(p) = \delta^{AB} \frac{i}{p^2 - M_A^2} \left(\eta_{\mu\nu} - \frac{p_\mu p_\nu}{M_A^2} \right).$$



(a)



(b)



(c)



(d)

- One loop scalar mass corrections

$$-i\Sigma_a(0) = g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{n}{k^2}$$

$$-i\Sigma_b(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \left(\frac{n-1}{k^2 - M_A^2} - \frac{1}{M_A^2} \right)$$

$$-i\Sigma_c(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{1}{k^2}$$

$$-i\Sigma_d(0) = -g^2 C_2(N) \int \frac{d^n k}{(2\pi)^n} \frac{1}{M_A^2}.$$

-

$$\delta m_h^2 \sim \frac{g^2 M_{\text{LW}}^2}{16\pi^2} \ln \left(\frac{\Lambda^2}{M_{\text{LW}}^2} \right).$$

- Mixing between LW and massive gauge bosons

$$\mathcal{L}_{\text{mix}} = M_W^2 (W_\mu^+ \tilde{W}^{-\mu} + \tilde{W}_\mu^+ W^{-\mu}) + M_Z^2 Z_\mu (\cos \theta_W \tilde{W}^{3\mu} - \sin \theta_W \tilde{B}^\mu)$$

$$\Delta\rho = \rho - 1 = -\frac{\sin^2 \theta_W M_Z^2}{M_1^2}.$$

$|\Delta\rho| < 10^{-3}$ leads to $M_1 > 1\text{TeV}$.

Fermions

-

$$\mathcal{L}_{\text{hd}} = \bar{Q}_L i \hat{\mathcal{D}} Q_L + \frac{1}{M_Q^2} \bar{Q}_L i \hat{\mathcal{D}} \hat{\mathcal{D}} \hat{\mathcal{D}} Q_L.$$

- Eliminate the higher derivative term by introducing LW-quark doublets $\tilde{Q}_L, \tilde{Q}'_R$ which form a real representation of the gauge groups. Lagrangian in this formulation becomes

$$\begin{aligned} \mathcal{L} = & \bar{Q}_L i \hat{\mathcal{D}} Q_L + M_Q (\bar{Q}_L \tilde{Q}'_R + \bar{Q}'_R \tilde{Q}_L) \\ & + \tilde{Q}_L i \hat{\mathcal{D}} Q_L + \bar{Q}_L i \hat{\mathcal{D}} \tilde{Q}_L - \bar{Q}'_R i \hat{\mathcal{D}} \tilde{Q}'_R. \end{aligned}$$

Remove LW-fermions with their equations of motion

$$\tilde{Q}'_R = -\frac{i \hat{\mathcal{D}}}{M_Q} Q_L, \quad \tilde{Q}_L = \frac{\hat{\mathcal{D}} \hat{\mathcal{D}}}{M_Q^2} Q_L,$$

- To diagonalize the kinetic terms, we introduce the shift $\hat{Q}_L = Q_L - \tilde{Q}_L$, and the Lagrangian becomes

$$\mathcal{L} = \overline{Q}_L i \not{D} Q_L - \overline{\tilde{Q}}_L i \not{D} \tilde{Q}_L - \overline{\tilde{Q}'_R} i \not{D} \tilde{Q}'_R + M_Q (\overline{\tilde{Q}}_L \tilde{Q}'_R + \overline{\tilde{Q}'_R} \tilde{Q}_L) - \overline{Q}_L \gamma_\mu \tilde{A}^\mu Q_L + \overline{\tilde{Q}}_L \gamma_\mu \tilde{A}^\mu \tilde{Q}_L + \overline{\tilde{Q}'_R} \gamma_\mu \tilde{A}^\mu \tilde{Q}'_R.$$

- New FCNC but small.

Conclusion

PLEASE LET THE LHC WORK