CONNECTING STRING THEORY TO THE REAL WORLD

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String theory became a hot subject in the mid-1980s when it became clear that it might give a deeper understanding of the origins of the standard model and a consistent quantum theory containing gravity.

At that time five consistent string theories were known, each of which requires ten spacetime dimensions and supersymmetry.

Type I, Type IIA, Type IIB

$SO(32)$ heterotic, $E_8 \times E_8$ heterotic

Each of these theories is entirely free of adjustable dimensionless parameters. All dimensionless parameters arise

- dynamically as the expectation values of scalar fields

  or

- as integers that count something such as topological invariants, physical objects (branes), or quantized fluxes.

One scheme looked particularly promising. Specifically, the $E_8 \times E_8$ heterotic theory has consistent vacuum solutions in which six spatial dimensions form a compact Calabi–Yau manifold, which has SU(3) holonomy, and the other four dimensions form Minkowski spacetime. Thus,

$$\mathcal{M}_{10} = \mathrm{CY}_6 \times M_{3,1}.$$

The effective four-dimensional theories at low energies have the following attractive features:

- They have the structure of supersymmetric grand unified theories. The advantages of low-energy supersymmetry and grand unification are therefore naturally incorporated.
- Each solution has a definite number of families of quarks and leptons determined by the topology of the CY space.

- The standard model gauge symmetry is embedded in one E_8 factor, and there is a hidden sector, associated to the second E_8 factor. Supersymmetry can break dynamically (e.g., by gluino condensation) in the hidden sector. This breaking is communicated gravitationally to the visible sector.
- There are several good dark-matter candidates: The LSP (a neutralino or gravitino), axions, and hidden-sector particles.

Despite the excitement, the successes were qualitative, and there were many problems and puzzling questions.

- Hundreds of Calabi–Yau manifolds were known (now many thousands). Which one of them, if any, is the *right one*? Are there other schemes for getting quasi-realistic solutions?
- Why are there four other consistent superstring theories? Can they give interesting solutions, too?
- The CY compactification scenario described above is based on perturbation theory, but there is no good rea-

son to believe that the string coupling is small. What new nonperturbative features appear at strong coupling?

- These solutions give massless scalars (called moduli). They have gravitational strength interactions and are ruled out by standard tests of GR. How can we get rid of them?
- What ensures that the vacuum energy density (or dark energy) is sufficiently small, namely of order 10^{-120} in Planck units?
- What does string theory have to say about cosmology?

Lessons of the Second Superstring Revolution

In the mid-1990s there was a remarkable burst of progress in addressing some of these issues. The main lessons were the following:

There is just one theory! What had been viewed as five theories are actually five different corners of a space of solutions to a unique underlying theory. The five superstring theories are related by various dualities:

T-duality ($R \rightarrow 1/R$) relates the two type II superstring theories, and also the two heterotic string theories.

S-duality ($g_s \to 1/g_s$) relates the type I superstring theory and the $SO(32)$ heterotic string theory.

The type I theory can be derived from the type IIB theory by a procedure called orientifold projection.

New stable objects, called p-branes, arise nonperturbatively. They carry conserved charges and satisfy generalized Dirac quantization conditions. The main categories are D-branes, M-branes, and NS5-branes. NS5-branes are magnetic duals of fundamental strings.

D-branes are characterized by Dirichlet boundary conditions for open strings. This has the crucial consequence that the world-volume theories of Dp-branes are Yang–Mills gauge theories in $p + 1$ dimensions. p takes even values in the type IIA theory and odd values in the type IIB theory.

A system of N coincident Dp-branes gives a $U(N)$ gauge theory. Other gauge groups can also be achieved if the D-branes coincide with certain types of singularities.

The strong-coupling limit of the Type IIA superstring theory or the $E_8 \times E_8$ heterotic string theory gives 11-dimensional M-theory.

In the $E_8 \times E_8$ heterotic case the 11th dimension is a line interval, so the 11-dimensional spacetime has two 10-dimensional boundaries. One set of E_8 gauge fields is localized on each boundary.

After Calabi–Yau compactification, it is possible for the volume of the CY manifold to vary along the 11th dimension, which opens up new possibilities for phenomenology.

Intersecting D-brane Models

The fact that D-branes carry Yang–Mills gauge theories suggests another approach to model-building based on type II superstrings. One again starts with a compactification of the form $K_6 \times M_{3,1}$. Then one introduces D$(3+n)$-branes that wrap n-dimensional cycles of K_6, while the other dimensions are spacetime filling.

For example, consider D6-branes in the type IIA theory compactified on a six-torus. Suppose that a set of N_1 D6-branes wraps three compact dimensions and a second set of N_2 D6-branes wraps a different three-cycle. The two stacks of branes give a $U(N_1) \times U(N_2)$ gauge theory in four dimensions.

Open strings connecting the two stacks of branes at their intersection points give chiral matter transforming as (N_1, N_2). This is reminiscent of the standard model and many of its possible extensions.

There are many schemes of this type. The successes and difficulties in this approach are comparable to those of CY compactification of the heterotic string.

Flux compactifications

Type II superstrings contain various massless n-form gauge fields (in the RR sector)

$$A_n = \frac{1}{n!} A_{\mu_1 \mu_2 \ldots \mu_n} dx^{\mu_1} \wedge dx^{\mu_2} \wedge \ldots \wedge dx^{\mu_n}.$$

n is odd in the type IIA theory and even in the type IIB theory. These have gauge-invariant field strengths of the form

$$F_{n+1} = dA_n,$$

generalizing Maxwell theory ($n = 1$). D-branes are sources for these fields.

There is also a two-form B_2 (in the NS sector) for which the fundamental strings are electric sources and NS5-branes are magnetic sources.

If the compact dimensions contain nontrivial $(n+1)$-cycles, C_{n+1}, it is possible to have (quantized) flux threading the cycle:

$$\int_{C_{n+1}} F_{n+1} = 2\pi N.$$

Such possibilities greatly increase the number of possible quantum vacua, though there are constraints that must be satisfied. Mike Douglas has analyzed one particular CY compactification of the type IIB theory and estimated the number of distinct flux vacua to be about 10^{500}.

Warped Compactification

One of the important properties of flux compactifications is that they give rise to warped geometries. This means that the ten-dimensional geometry is no longer a direct product of the form

$$\mathcal{M}_{10} = K_6 \times M_{3,1}.$$

Instead, the 4d Minkowski metric part of the 10d metric is multiplied by a warp factor $h(y)$ that depends on the position y in the internal manifold

$$ds_{10}^2 = h(y) dx \cdot dx + ds_6^2.$$

In the brane-world picture one can have spacetime-filling D3-branes that are localized in the internal manifold. Randall and Sundrum (hep-ph/9905221) have proposed that a large ratio between the warp factors at the position of a standard model brane and a Planck brane could provide a solution of the hierarchy problem. The RS scenario can be made rather precise in the context of flux compactifications with branes added.

Moduli Stabilization and the Landscape

The moduli problem — the occurrence of massless scalar fields ϕ_i with continuously adjustable vacuum expectation values — can be solved in the context of flux compactification. The fluxes induce a nontrivial potential energy function for the moduli $V(\phi_i)$. This landscape has isolated minima, which are what Douglas counted. The moduli are massive at a minimum.

There are many issues. For example, one of the moduli fields is the dilaton Φ, whose vacuum value determines the string coupling constant:

$$g_{\mathrm{s}} = \langle e^{\Phi} \rangle.$$

If this is stabilized at a nonzero value, then perturbation theory does not make sense. We have few other tools for studying solutions, so this is a serious problem.

The proliferation of vacua raises many questions: Is it completely hopeless to find the right one? Is it meaningful or useful to assign probabilities and study the vacua statistically? One proposal is that a probability distribution is determined by the wave function of the Universe.

The Cosmological Constant

Perhaps the values of the cosmological constant in the various vacua are randomly distributed over a range of order unity (in Planck units). Then, about 1 in 10^{120} would have roughly the right value. If there are 10^{500} or more vacua, this is a very large number even though it is a very small fraction.

There has been a great deal of discussion about this. I will restrain myself from saying more.

String Cosmology

There is a lot of evidence that the very early Universe underwent a period of inflation during which the scale factor grew exponentially by a factor of at least e^{60}. In simple field theory models this is described in terms of a "slowly rolling" scalar field called an inflaton.

Proposals for the string-based origins of inflation, or possible alternatives, are being explored extensively. Let me sketch a specific scenario due to Kachru et al. (hep-th/0301240 and 0308055). It takes place in the setup of

CY compactification with flux and warped throats in the geometry.

Inflation takes place as a D3-brane moves down a throat, attracted to an anti-D3-brane at the bottom until they collide and annihilate. A scalar mode of an open string connecting the branes is the inflaton.

The annihilation releases the brane tension energy. It heats up the Universe to start the hot big bang epoch. All sorts of strings are produced, and some might survive to be observable as cosmic superstrings.

Is String Theory the Only Quantum Theory of Gravity?

This section is based on arXiv:0704.0777 [hep-th] by Michael Green, Hirosi Ooguri, and JHS.

Recently, there has been some speculation that four-dimensional $\mathcal{N} = 8$ supergravity might be ultraviolet finite to all orders in perturbation theory (Green et al., Bern et al.). If true, this would raise the question of whether $\mathcal{N} = 8$ supergravity might be a consistent 4d quantum theory that is an alternative to string theory.

A related question is whether $\mathcal{N} = 8$ supergravity can be obtained as a well-defined limit of superstring theory. I will argue that such a supergravity limit of string theory does not exist, irrespective of whether or not the perturbative approximation is free of ultraviolet divergences.

We will study limits of Type II superstring theory on T^6. The analysis is analogous to the study of the decoupling limits of Dp-branes, where field theories on branes decouple from closed string modes in the bulk (Sen, Seiberg).

Perturbative spectrum

It is sufficient to consider a torus that is product of six circles of radius R. Numerical factors, which are irrelevant to the discussion, will be dropped.

We are interested in whether there is a limit of string theory that reduces to $\mathcal{N} = 8$ supergravity. In other words, we require the decoupling of all excited string states, together with the Kaluza–Klein excitations and string winding states associated with the 6-torus.

A necessary condition for this decoupling is that there is a limit in which these states are all infinitely massive compared to the Planck scale ℓ_4. This is achieved by taking

$$\frac{1}{R}, \quad \frac{1}{\ell_{\rm s}}, \quad \text{and} \quad \frac{R}{\ell_{\rm s}^2} \gg \frac{1}{\ell_4}\,,$$

with ℓ_4 fixed. Then the surviving perturbative states are the 256 massless states of maximal supergravity, which is $\mathcal{N} = 8$ supergravity when $d = 4$.

Nonperturbative spectrum

Let us now consider the spectrum of half-BPS nonperturbative superstring excitations in this limit. In order to obtain the pure $\mathcal{N} = 8$ supergravity in four dimensions, these nonperturbative states also need to decouple, so their masses must also satisfy $M \gg 1/\ell_4$.

Kaluza–Klein and wrapped-string states are half-BPS objects that carry a conserved electric charge. In four dimensions their magnetic duals are stable half-BPS nonperturbative states.

The BPS saturation condition together with the Dirac quantization condition implies quite generally that the masses of a BPS particle and its magnetic dual are related by

$$M \cdot \tilde{M} \sim \frac{1}{\ell_4^2}.$$

Thus there is no limit in which all BPS particles become much heavier than the Planck scale. In particular, the magnetic duals of Kaluza–Klein excitations (Kaluza–Klein monopoles) are BPS states with $M \sim R/\ell_4^2 \to 0$.

The reappearance of ten dimensions

There is an infinite number of light KK monopoles, since each of the six magnetic charges can be an arbitrary integer.

Among the elements of the U-duality group of the toroidally compactified string type II string, $E_7(\mathbb{Z})$, is the S-duality transformation that interchanges the electric charges with the corresponding magnetic charges.

This S-duality is described by the following transformations of the moduli:

$$\mathrm{S}: \;\; R \to \tilde{R} = \frac{\ell_4^2}{R} \;\text{ and }\; \ell_\mathrm{s} \to \tilde{\ell}_\mathrm{s} = \frac{\ell_4^2}{\ell_\mathrm{s}}.$$

Note that this transformation inverts the radius R in four-dimensional Planck units (in contrast to T-duality, which inverts R in string units). It also implies that the string coupling constant is inverted.

Thus, in the dual frame in which the compactification scale $\tilde{R} \to \infty$, the six-torus is decompactified.

The fact that an infinite set of states from the nonperturbative sector become massless shows that the limit of interest does not result in pure $\mathcal{N} = 8$ supergravity in four dimensions. Rather, it results in 10-dimensional decompactified string theory with the string coupling constant inverted.

These results illustrate the conjectures of Vafa and Ooguri about the geometry of continuous moduli parameterizing the string landscape. The conjectures concern consistent quantum gravity theories with finite Planck scale in four or more dimensions. Among the conjectures are the statements that, if a theory has continuous moduli, there are points in the moduli space that are infinitely far away from each other, and an infinite tower of modes becomes massless as a point at infinity is approached. A theory that omits these modes is said to be in the Swampland, which is bad.

Since the limit considered here corresponds to a point in the moduli space of string compactifications at infinite distance from a generic point in the middle of moduli space, the conjectures predict than an infinite number of particles become massless in the limit, as we have found.

Thus, $\mathcal{N} = 8$ supergravity is in the Swampland.

The original motivation of this work was to investigate the relation between superstring theory and $\mathcal{N} = 8$ supergravity and to explore under what conditions supergravity might be ultraviolet finite. We have not answered that question, but we have that there is no limit of toroidally compactified superstring theory in which the stringy effects decouple and only the 256 massless supergravity fields survive below the four-dimensional Planck scale.

Conclusion

Best wishes to the BCTP!